Resolving the Black Hole Information Paradox: Is the Universe a Hologram?

Recent works of science-fiction have attributed all manner of mystical effects to black holes. But what if some of this science-fiction was in fact reality?
With insight from black hole thermodynamics and string theory we explore the possibility that the universe is a hologram.

Background picture: Artist's depiction of a black hole.[a]

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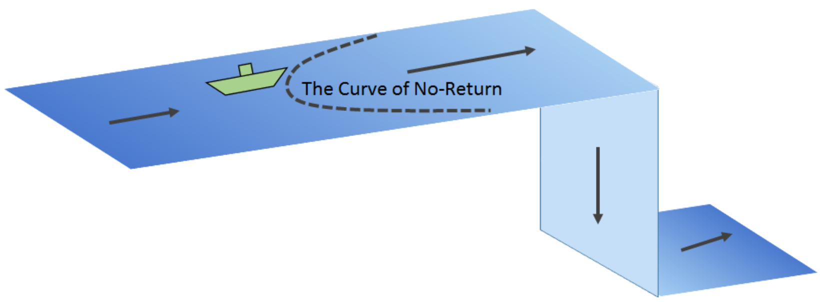
The Dead Star

Stars are the universe's source of light, but they do not last forever. When a star can no longer fuel the nuclear fusion within its core, it begins to collapse under its own gravity. If the star is sufficiently massive it will collapse into a black hole, an entity so dense that not even light can escape its gravitational pull.[1] What was initially a hot, fiery, giver of light has become an all-consuming, cold and dark place – or so it would seem.

Black holes are direct solutions of the Einstein-Maxwell equations of general relativity.[2] To solve the equations one need only specify the mass, charge and angular momentum of the black hole. This is known as the *no-hair theorem*[2] because it means black holes are almost featureless. The other features of bodies entering a black hole are irreversibly lost beyond the horizon.[3]

Physically speaking, a black hole corresponds to an infinitely dense singularity. Surrounding the singularity is the event horizon, of size determined by the Schwarzschild radius. The event horizon defines a spherical boundary after which light can no longer escape the black hole's pull.[3] In his theory of special relativity, Einstein showed that nothing can travel faster than the speed of light.[3] Thus the event horizon represents the point of no return, once you've crossed it you're destined to reach the singularity at the centre.

The theory of relativity does away with our intuitive sense that observations are objective. A person falling through an event horizon will not see the same thing as someone watching from afar. Meet Stacey and her friend Fred; Stacey is stationary relative to the black hole and watches Fred fall into the event horizon. As Fred crosses the event horizon nothing interesting will happen in his reference frame; the event horizon is not a physical entity, it is just a mathematically calculated boundary after which light can no longer escape*. However, because of gravitational time dilation, Stacey observes something drastically different. She sees Fred slow down more and more as he approaches the horizon. This is because Stacey seeing Fred involves light reflecting off him and coming back to her. As it moves closer to the event horizon light is 'pulled' more strongly by the black hole, making it take longer to return to Stacey. The light that hits Fred exactly as he crosses the horizon will take an infinite time to return, hence according to Stacey he never crosses the horizon.[5]



POINT OF NO RETURN: The event horizon of a black hole is akin to a boat travelling towards a waterfall. The current is greater closer to the waterfall. This means there will be a point, which once crossed, the boat will no longer be able to travel backwards and escape the drop (this is the curve of no-return). The sailors will not notice anything upon crossing this boundary, "it's not a *thing*, it's a *place*".[4]

Entropy & Information

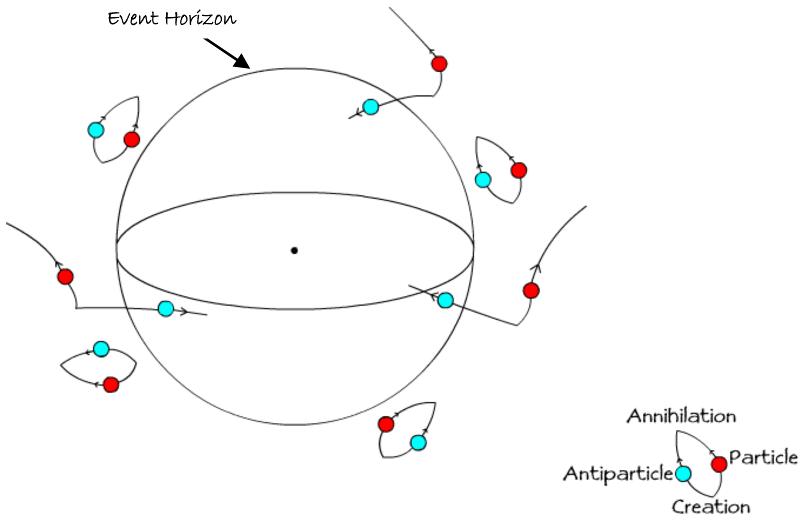
Thermodynamic entropy is a measure of the number of ways you can rearrange the 'ingredients' of system without altering its overall appearance.[13] A vase has a low entropy as it has a very specific design and structure. If the vase were to fall and break into pieces its structure would become disordered and multiple arrangements of its atoms could produce the same broken mess. This means a broken vase has a higher entropy.

The second law of thermodynamics states that the entropy of the universe can only ever increase.[7] The vase will not naturally fix itself. Even if one were to collect all the pieces and rebuild the vase, the entropy of the universe would still increase because of heat produced during the rebuilding process.

Shannon entropy[8] provides a way of defining the entropy of a system in terms of the information it contains, it is conceptually

* In reality Fred's body will be ripped apart by tidal forces and he'd have suffered death by spaghettification long before having reached the horizon.[3] Nonetheless, to help describe the physics we pretend that it is possible for an observer to stay alive until they reach the singularity.

equivalent to its thermodynamic counterpart. Information is stored in bits. This is not the colloquial bit used in everyday talk, it refers to a unit of information.[6] Take, for example, a box which either has or hasn't got a marble in it. The existence of the marble in the box represents one bit of information, a yes or a no. If you had a collection of these boxes then the information would be equal to the number of boxes, a yes or a no for each box.[9] In physics there are numerous variables to consider, such as position and velocity. Each of these variables adds to the amount of information a system contains.†



HAWKING RADIATION: Pair production at the event horizon of a black hole can result in one of the particles escaping while the other is consumed by the black hole. This is the mechanism by which black holes emit Hawking radiation.[15]

Black Hole Thermodynamics

It all started when Jacob Bekenstein, a professor from the University of Jerusalem, wondered what happens to the entropy of objects that enter black holes. By considering the effect of adding one bit of information at a time to a black hole, he found that the entropy of a black hole was proportional to the surface area of its event horizon.[6] The area here is measured in Planck units. One Planck area equals 2.6×10^{-70} square metres[10], an absolutely tiny amount!

This is a most peculiar result.[11] If you were to add water to a glass it would seem right to suggest that for every molecule of water that you add you're adding some information. Thus the entropy should be proportional to the volume of the water.[6] But in the case of a black hole, it turns out that its surface area increases in proportion to the amount of information that enters it!

In 1974, following the work of Bekenstein, Stephen Hawking discovered a mechanism whereby black holes emit radiation.[12] This was attributed to one of Physics' more mind-boggling phenomena, that vacuums are not truly empty.

According to quantum field theory a vacuum is buzzing with particles popping into and out of existence.[13] This is as a result of Heisenberg's Uncertainty Principle which states that *there are certain pairs of measurements that one cannot accurately measure simultaneously*.[13] Take for example position and momentum. To measure the position of a particle at a certain time one would have to shine light on the particle and then observe the reflection. The lower the wavelength of the light, the more accurate the measurement of position. However, this accurate position measurement comes at a cost. As you decrease the wavelength of a photon its energy increases. This makes it impart a larger impulse on the particle, moving the particle and creating an uncertainty in its momentum. So to measure momentum to a high precision, low energy photons are required. But to measure position to high precision, high energy is required.[13] You can't have both, thus one can never simultaneously know the exact position and momentum of a particle. In the jargon, position and momentum are said to be complementary.[6]

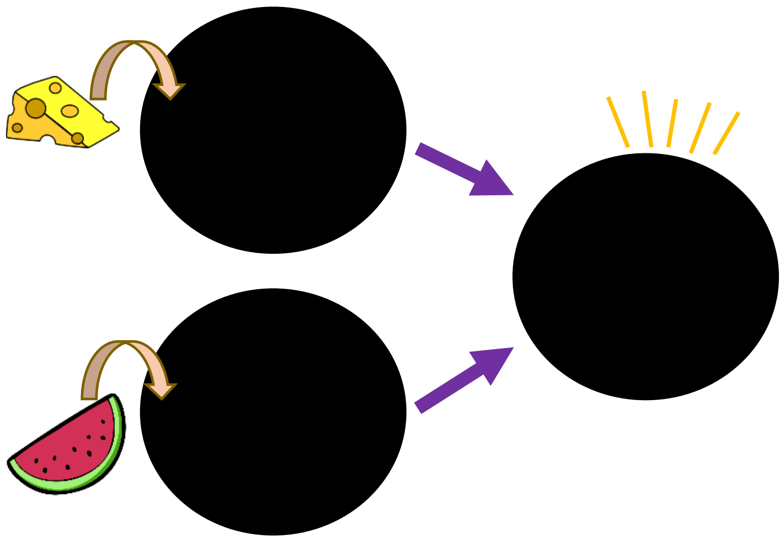
Another pair of complementary observables is energy and time.[3] This means that energy may spontaneously appear inside a vacuum for short periods of time. These fluctuations in energy can lead to the pair production of a particle and anti-particle. Pair production is a result of Einstein's famous equation, $E=mc^2$, which states that mass and energy are interchangeable. The pair do not

† For our purposes an increase in entropy corresponds to an increase in information. The two will be used interchangeably throughout the article.

last long however and annihilate almost immediately.[3]

When pair production occurs around the event horizon of a black hole it is possible that whilst one of the particles is pulled in, the other manages to escape. In this scenario the particles are unable to annihilate with each other and in order to conserve energy the black hole's mass decreases. A lower mass black hole has a smaller Schwarzschild radius meaning that black holes decrease in size and evaporate away as they emit Hawking radiation.[14]

Hawking went on to use a purely quantum mechanical argument to validate Bekenstein's entropy relation and found that the entropy of a black hole is equal to its surface area divided by 4.[16]



THE INFORMATION PARADOX: Regardless of what enters a black hole, mixed state Hawking radiation is the end product. This violates unitarity and means that information is destroyed in a black hole. (Note the circumference of the circles represents the event horizon.)[b],[c]

Where Did the Information Go?

The notion of Black Hole evaporation resulted in a plethora of unanswered questions. Firstly, what is this entropy? Entropy describes the information of a system, but if black holes have no hair then what information can it be describing?[6]

Moreover, when considering an evaporating black hole, Fred will continue to appear close to the event horizon until the black hole fully evaporates. At this point the black hole is no longer there to 'pull back' the light and Stacey will see him for one final time before he disappears.[5] But where did he go? If a black hole evaporates and eventually vanishes from existence, what happens to all the information it consumed during its lifetime? This would be no different to you putting food in a fridge, a special type of fridge that has a tendency to evaporate, and coming back to find that both the fridge and the food had vanished! Quantum mechanics demands that this cannot be, information can't just have disappeared.[6]

Let's discuss this second question more rigorously. Although quantum systems are innately probabilistic in nature, they do exhibit some sense of determinism. In quantum mechanics the state of a system is described by a wavefunction, containing everything there is to know about the state. The wavefunction evolves over time in a *unique* way. This means that given the state of a system at a particular time, one can work out what state the system will be in at a certain time in the future and also what state it was in in the past. It follows that you cannot have two different quantum states evolving into the same state as this would mean there is no unique past, or future, state.‡ This is known as *unitarity* and must be obeyed according to quantum mechanics.[6],[17]

For example, consider a tub of water.[6] If it were to be heated up and evaporate, one could technically trace back every evaporated gas molecule to where it was initially in the tub. This ability to trace back is called micro-reversibility[9]. The motion of each gas molecule is a direct result of the heating of the water, hence information is conserved and unitarity is obeyed.

On the other hand, Hawking radiation occurs as a result of random quantum fluctuations that are independent of the black information entering the black hole. Moreover, light cannot escape the event horizon. Thus

‡ Note that this is only true if we do not look at or disturb the system. In quantum mechanics a wavefunction collapses under measurement and all of its history is lost; particles do not respond well to being looked at!

INFORMATION EXPOSED: According to string theory all matter is fundamentally made up of vibrating strings (1). As a string approaches the event horizon the vibrations slow down, it stretch out and the information it contains is exposed (2), (3).[9]

once a body has passed through, it is not possible for it to transmit information back to the horizon. So there is no way that the Hawking radiation can contain, or become entangled with, the information inside the black hole.[18] This means that one can throw a piece of cheese into one black hole and a watermelon into another, once the black hole has evaporated you get the same end result, random Hawking radiation.

This is in clear violation of unitarity but it did not bother Hawking. In 1976 he published a paper[19] in which he stated that black holes do indeed destroy information. This disturbed many physicists, most notably Gerard 't Hooft and Leonard Susskind, the second of whom declared war with Hawking. He did however ensure that this "war" was purely academic.[6]

"God not only plays dice, He sometimes throws the dice where they cannot be seen."[19] – Stephen Hawking 1976

Resolving the Paradox

The race to find the missing information had begun with numerous theories being proposed. Some suggested that black holes never truly evaporate and that they leave behind Planck-sized remnants containing all of the information.[20] Others suggested that the information was stored in baby-universes.[21]

Susskind had other ideas. He mused that the information must somehow be extracted at the horizon and encoded into the Hawking radiation. However it could not simply be copied, as that would violate the *no-cloning theorem* of quantum mechanics[22]. Just as information cannot be destroyed, it cannot be duplicated. It wasn't until the 90s that he realised string theory could resolve the paradox.[6]

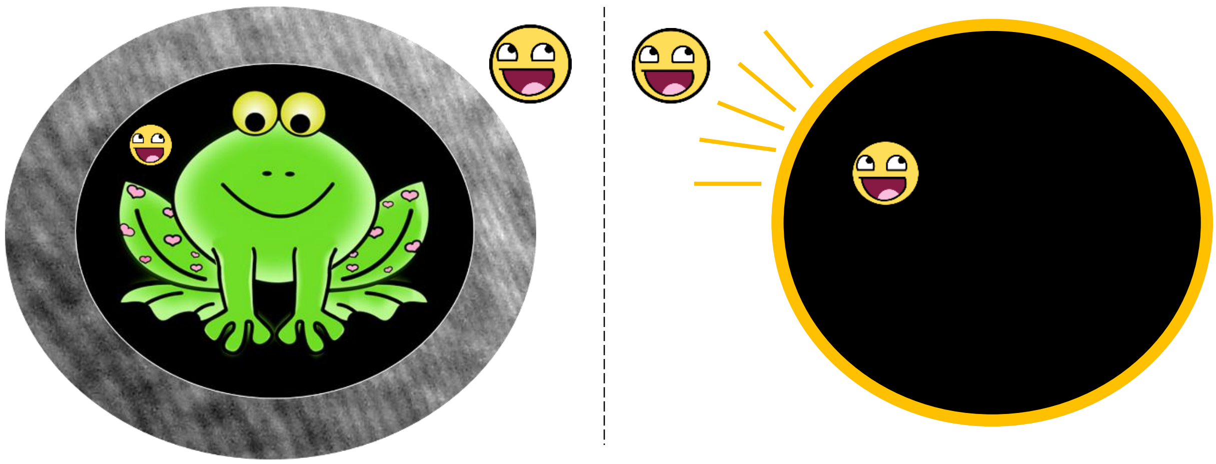
String theory is Physics' best attempt to merge general relativity with quantum mechanics. It suggests that every particle is fundamentally made up of vibrating one dimensional strings. The way in which the strings vibrate determines all of the particle's properties. Hence the information of all matter is stored within its constituent strings.[13]

Let's revisit Fred and Stacey. Say Fred was a highly trained radiographer and took an X-Ray machine with him on his journey towards the black hole. This time Fred emitted a continuous beam of X-Rays as he approached the event horizon. X-Rays are not visible to the human eye, they have too low a wavelength, so initially Stacey will see none of this radiation. However as the source moves towards the black hole, gravity starts to stretch out the wavelength of the X-Rays and Stacey will begin to see violet light. This is known as gravitational red shift.[3] The colour will then change, going through the full visible spectrum, until the wavelength becomes too large for Stacey's eyes to detect.

The effect of a black hole's gravity on the wavelength of light is analogous to its effect on strings. As the strings get closer to the event horizon they grow and spread out until they eventually engulf the black hole. Hence the entire history of the black hole, including everything that went into it, is smeared across its event horizon. What was a bald, cold spherical shell has become a "hot soup"[6] of strings.

So according to string theory Fred will pass through the event horizon undisturbed, as before, and will see his own information directly in 3D.§ However Stacey will now see all of Fred's information spread out across a 2D membrane formed by the strings surrounding the event horizon. Also, the presence of this membrane allows the Hawking

§ This postulate is currently in question. A group of researchers have suggested that the event horizon actually takes the form of a firewall in the free falling reference frame. In which case Fred would be thermalised as he falls through.[28]



HOLOGRAPHIC BLACK HOLES: **The left hand side represents a spherical hologram. An observer inside the sphere will see the 3D image of the hologram, a frog. The observer outside only sees the 2D holographic film coating the sphere, however both observers see the same information.**

This is analogous to a black hole, right hand side, in which the falling observer sees his own information in 3D and the outside observer sees the 2D encoding of the information spread over the event horizon and eventually emitted as Hawking radiation.

Hence a black hole behaves as a hologram in the sense that its 3D interior is described by the information contained on its 2D event horizon.[24],[d],[e]

radiation to become encoded with the information that enters the black hole. So as the black hole evaporates Stacey sees Fred's information in the form of Hawking Radiation.[6]

At first glance it seems that this is describing information that has been duplicated, contradicting the no-cloning theorem. This however is not the case. Just as the Uncertainty Principle states that two observables can be complementary, so too here both points of view are complementary. Stacey cannot wait outside the event horizon, collect the Hawking radiation of Fred's information (2D Fred) and then venture into the black hole to see 3D Fred. By the time she's collected the Hawking radiation, Fred will already have hit the singularity and be destroyed. This is known as black hole complementarity[16] and it means that 2D and 3D Fred are the exact same information just displayed in different forms. Both views are one and the same.[6]

A final achievement of this resolution is that the vibrations of strings provides an explanation for the source of black hole entropy.[23]

The Holographic Principle

A hologram is a 2D film encoded with the information necessary to produce a 3D image when exposed to light. The 2D encoding contains the same information as the 3D image, since if one knew how to translate the film one would be able to deduce the exact image.[24]

To understand the jump from black holes to the entire universe we must perform a thought experiment. Consider any object (that isn't a black hole) surrounded by a solid spherical shell. If the shell is continuously compressed, the density increases and it will eventually collapse into a black hole. The second law of thermodynamics tells us that the entropy of the black hole must be greater than or equal to the entropy of the original object. But we know that the entropy of a black hole is equal to its surface area**. Thus the entropy of a black hole defines a maximum entropy and a maximum information content for all matter.[6],[25]

This is somewhat ironic as originally it was thought that black holes had no entropy, however they now define a maximum entropy. Because of this, the information of all matter is limited by its surface area and so can be thought of as a hologram!

Is This Really True?

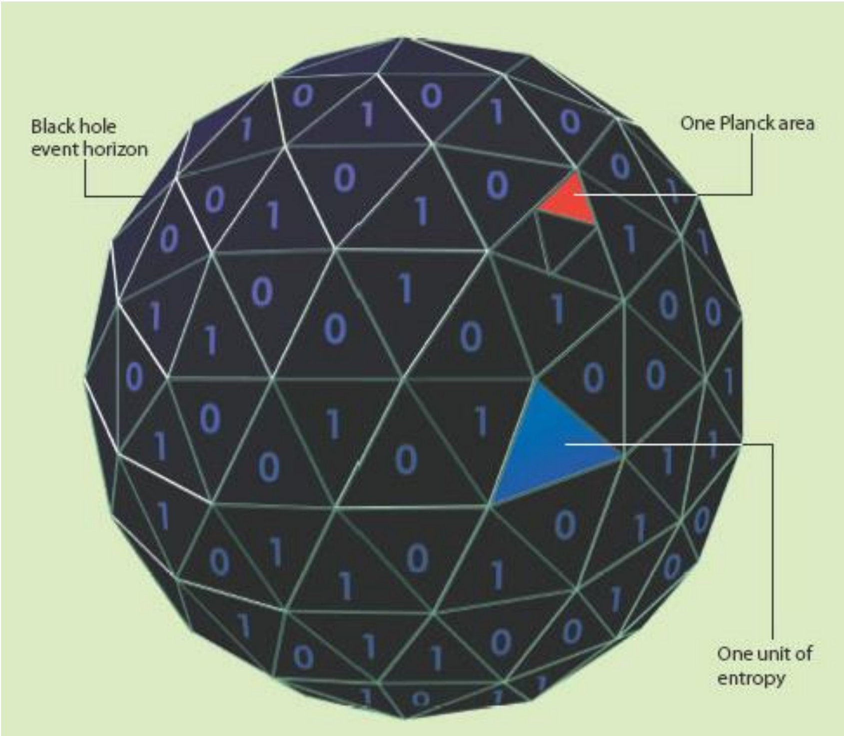
Well the honest answer is no one knows. The Planck length (approximately 10^{-35}m)[10], on which

** The proportionality constant is not important, it would simply define the size occupied by a single bit.[6]

information is allegedly encoded, is over a billion billion times smaller than the classical radius of an electron (approximately 10^{-15}m)[26]. Thus even modern-day particle detectors are nowhere near being able to resolve distances on the Planck scale. So direct observation seems to be out of the question for now.

At the Relativistic Heavy Ion Collider (RHIC) in Brookhaven National Laboratory[27] ions are smashed into each other to create “imaginary” black holes. Although these imaginary black holes do not exhibit the same star-swallowing gravitational strengths of a real black hole, which is lucky for us, they do emit thermal particles similar to Hawking radiation.[27] The imaginary black holes only last for around 10^{-23} seconds[27], but they are still the most promising potential source of experimental evidence.

There is also some evidence in the form of theoretical physics. A 5-dimensional anti–de Sitter spacetime described by string theory has been shown to be physically equivalent to a quantum field theory operating on its boundary.[11] Perhaps similarly our 4D spacetime could be the result of some 3D spacetime on a distant boundary.



INFORMATION ACCOUNTED FOR: The black hole’s entire history is encoded on its event horizon. Each bit (0 or 1) occupies one unit of entropy, 4 times the Planck area.[8]

Physics has a tendency to challenge our intuition. Relativity unified spacetime and quantum mechanics told us our world is probabilistic. Currently, general relativity and quantum mechanics are incompatible. In order to fully understand the universe the two must be unified into a theory of quantum gravity[9] and perhaps the holographic principle is one of the many steps on the way to achieving this revolution.

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